Diploma Programme
Programme du diplôme
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Mathematics
Higher level
Paper 3 – discrete mathematics

Thursday 21 November 2019 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 10]

A driver needs to make deliveries to five shops A, B, C, D and E. The driver starts and finishes his journey at the warehouse W. The driver wants to find the shortest route to visit all the shops and return to the warehouse. The distances, in kilometres, between the locations are given in the following table.

	A	***B***	***C***	***D***	***E***	***W***
A	-	11	28	15	20	40
B	11	-	25	20	32	36
C	28	25	-	16	22	39
D	15	20	16	-	12	42
E	20	32	22	12	-	41
W	40	36	39	42	41	-

(a) By deleting W, use the deleted vertex algorithm to find a lower bound for the length of a route that visits every shop, starting and finishing at W. [6]

(b) Starting from W, use the nearest-neighbour algorithm to find a route which gives an upper bound for this problem and calculate its length. [4]

2. [Maximum mark: 15]

(a) (i) State Fermat's little theorem.

(ii) Find the remainder when 15^{1207} is divided by 13. [7]

In parts (b) and (c), $(abc\ldots)_n$ denotes the number $abc\ldots$ written in base n, where $n \in \mathbb{Z}^+$. For example, $(359)_n = 3n^2 + 5n + 9$.

(b) Convert $(7A2)_{16}$ to base 5, where $(A)_{16} = (10)_{10}$. [4]

(c) Consider the equation $(1251)_n + (30)_n = (504)_n + (504)_n$.

Find the value of n. [4]

3. [Maximum mark: 6]

A linear recurrence relation is defined by

$$u_n = au_{n-1} + b\text{, where } a, b \in \mathbb{R}.$$

The first two terms are $u_1 = 25$ and $u_2 = 16$.

(a) Given that $\lim_{n\to\infty} u_n = 10$, show that $a = \frac{2}{5}$. [4]

(b) Hence find the value of u_3. [2]

4. [Maximum mark: 14]

(a) G is a simple, connected graph with eight vertices.

(i) Write down the minimum number of edges in G.

(ii) Find the maximum number of edges in G.

(iii) Find the maximum number of edges in G, given that G contains an Eulerian circuit. [5]

(b) H is a connected, planar graph, with v vertices, e edges and f faces. Every face in H is bounded by exactly k edges.

(i) Explain why $2e = kf$.

(ii) Find the value of f when $v = 9$ and $k = 3$.

(iii) Find the possible values of f when $v = 13$. [9]

5. [Maximum mark: 5]

Ten points are placed anywhere inside or on the perimeter of a square of side length 1.

Use the pigeon-hole principle to prove that at least two of these points have a distance between them that is less than or equal to $\frac{\sqrt{2}}{3}$.